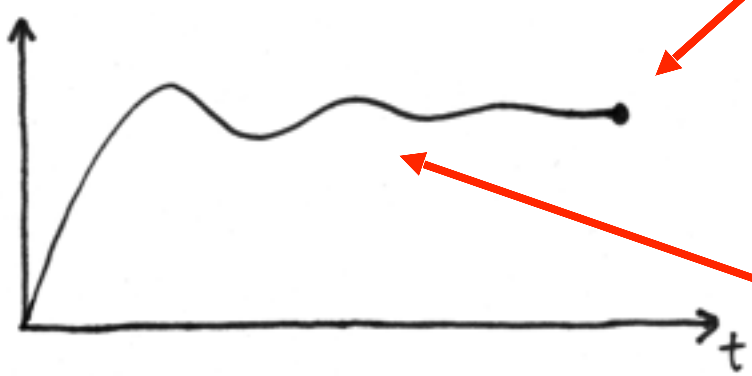
Dynamic models

Dynamic models

Dynamic models are often not difficult to formulate, but can be difficult to solve analytically.

Things that can usually be done quite easily:

Stable state analysis: we determine the final value in the long run



Simulation: we numerically determine the solution

Example of stable state analysis

Where are the equilibrium points of the differential equation?

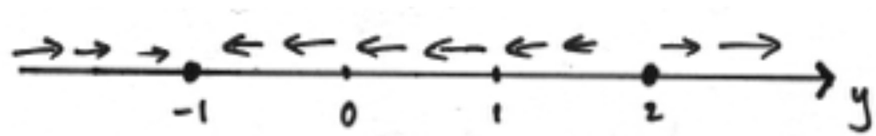
$$y' = (y+1)(y-2)$$

Set $y' = 0 \implies$

$y = -1$ and $y = 2$

Are these points stable?

We investigate the derivative along the phase line:



Stable point (a small disturbance gives a return to the equilibrium)

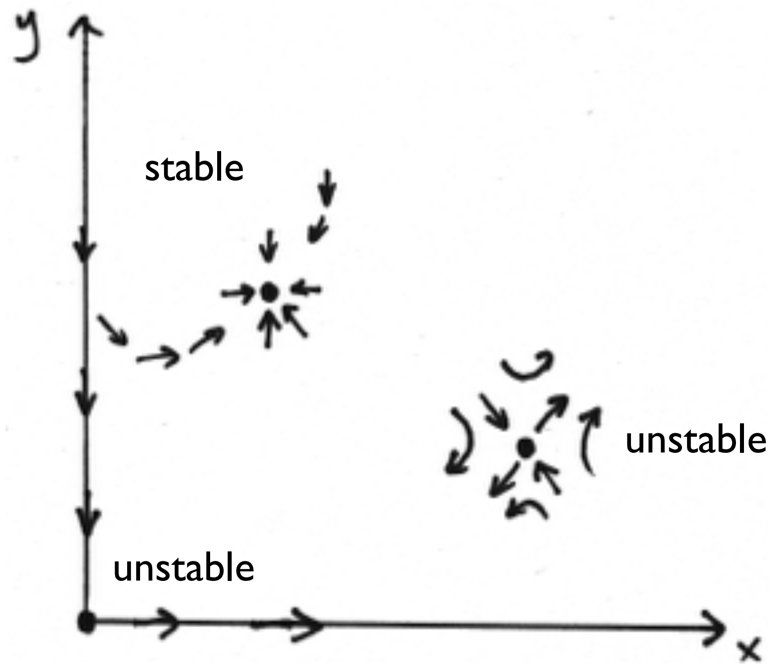
Unstable point (a small disturbance gives a movement away from the equilibrium)

Stable state analysis in two dimensions

$$x' = g(x,y)$$
$$y' = h(x,y)$$

For equilibrium points:
set $x'=0$, $y'=0$

phase plane analysis:



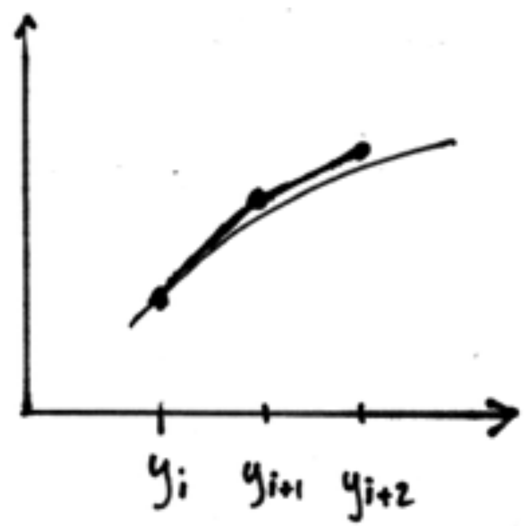
There are also more complicated cases!

How simulate?

Make time discrete in small intervals and use Eulers method:

$$y' = h(y, t)$$

$$y(t+\Delta t) = y(t) + h(y(t), t) \cdot \Delta t$$



More exact: Runge-Kutta method evaluates h in four points per iteration.

Standard form for multidimensional ODE´s

$$x_1' = g_1(x_1, x_2, \ldots, x_n, t)$$
$$x_2' = g_2(x_1, x_2, \ldots \qquad)$$
$$x_3' =$$
$$\vdots$$

If t is not an argument in g the system is called *autonomous*. This is natural for many systems.

Eulers method:

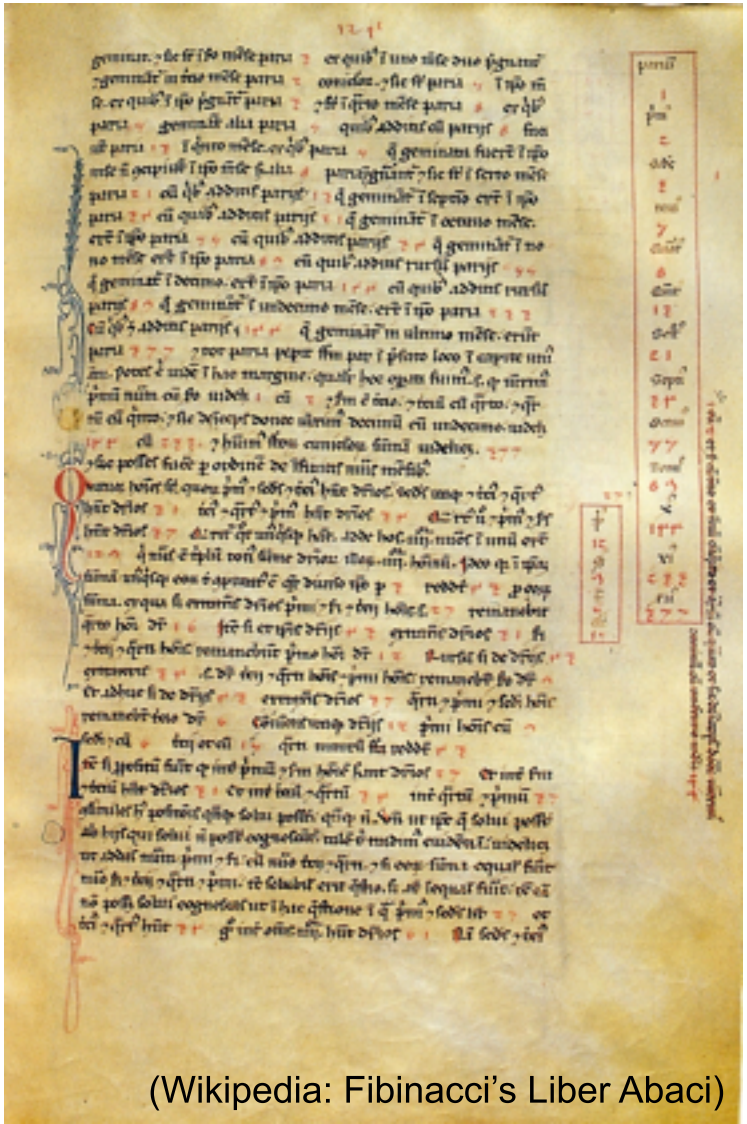
$$x_1(t+\Delta t) = x_1(t) + g_1(x_1, x_2, \ldots) \cdot \Delta t$$
$$x_2(t+\Delta t) = x_2(t) + g_2(x_1, x_2, \ldots) \cdot \Delta t$$
$$\vdots$$

Easy also to simulate multidimensional systems!

Difference Equations: ex Fibonacci sequence

$$F_n = F_{n-1} + F_{n-2},$$

$$F_0 = 0, \quad F_1 = 1,$$



(Wikipedia: Fibinacci's Liber Abaci)