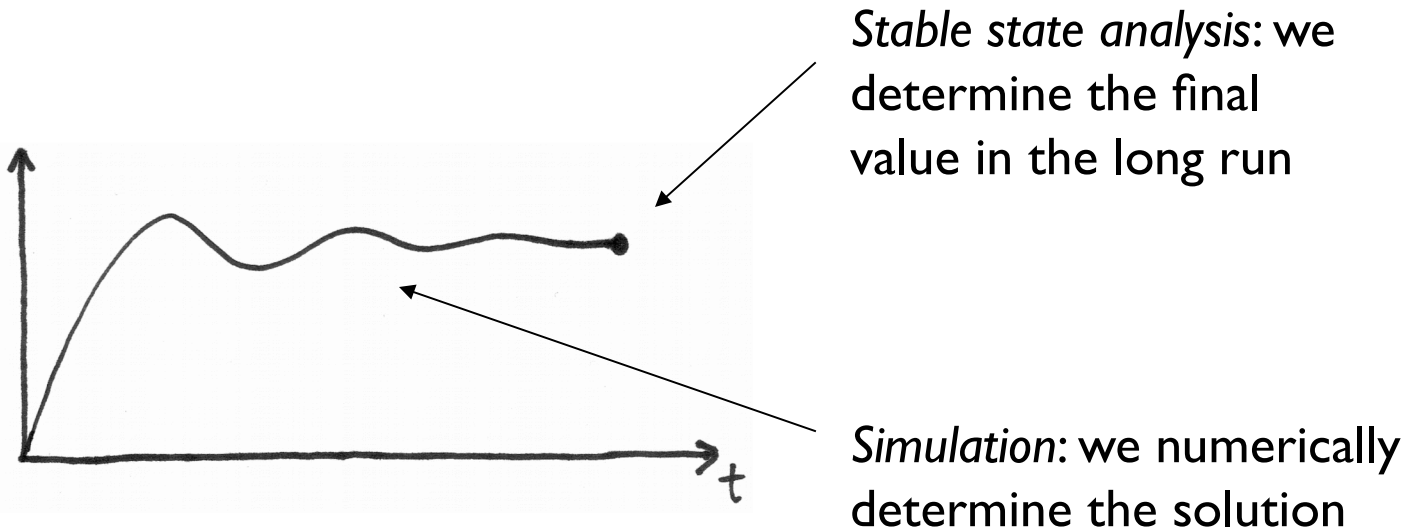


Dynamic models

Dynamic models

Dynamic models are often not difficult to formulate, but can be difficult to solve analytically.

Things that can usually be done quite easily:



Example of stable state analysis

Where are the equilibrium points of the differential equation?

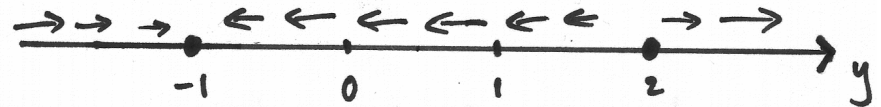
$$y' = (y+1)(y-2)$$

$$\text{Set } y' = 0 \Rightarrow$$

$$y = -1 \text{ and } y = 2$$

Are these points stable?

We investigate the derivative along the phase line:



Stable point (a small disturbance gives a return to the equilibrium)

Unstable point (a small disturbance gives a movement away from the equilibrium)

Stable state analysis in two dimensions

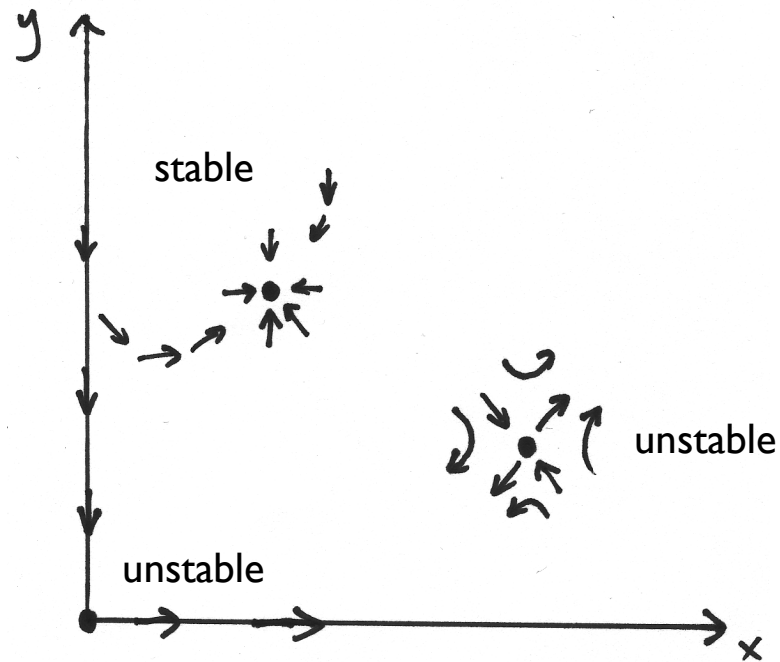
$$x' = g(x, y)$$

$$y' = h(x, y)$$

For equilibrium points:

$$\text{set } x' = 0, y' = 0$$

phase plane analysis:



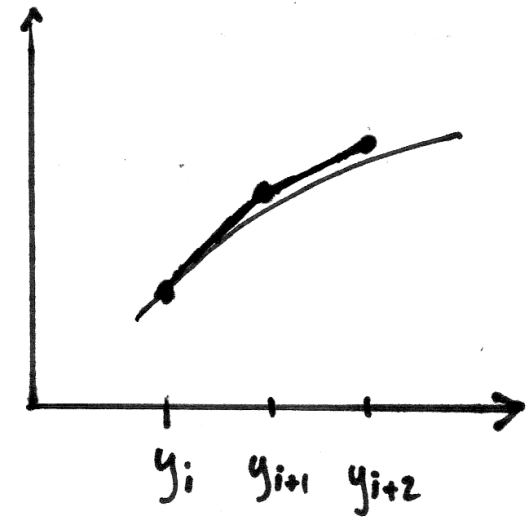
There are also more complicated cases!

How simulate?

Make time discrete in small intervals and use Eulers method:

$$y' = h(y, t)$$

$$y(t + \Delta t) = y(t) + h(y(t), t) \cdot \Delta t$$



More exact: Runge-Kutta method evaluates h in four points per iteration.

Standard form for multidimensional ODE's

$$x_1' = g_1(x_1, x_2, \dots, x_n, t)$$

$$x_2' = g_2(x_1, x_2, \dots)$$

$$x_3' =$$

⋮

If t is not an argument in g the system is called *autonomous*. This is natural for many systems.

Eulers method:

$$x_1(t + \Delta t) = x_1(t) + g_1(x_1, x_2, \dots) \cdot \Delta t$$

$$x_2(t + \Delta t) = x_2(t) + g_2(x_1, x_2, \dots) \cdot \Delta t$$

⋮

Easy also to simulate multidimensional systems!