

Optimization problems

Meet your Course Representatives

From Chalmers:

- Filip Andréasson
- Isabelle Ermeryd Tankred
- Jonas Högne
- Jonathan Köre
- Markus Pettersson
- Carolina Larsson
- Lina Turner

From GU:

- Sophia Thanh Pham
- Emaniuel Olaison
- David Hultsten
- Karl Andersson

What is an optimization problem?

Example: shortest path problem

Minimize

the length

over

all paths from a to b in a graph

Minimize

the objective function

over

the set of feasible solutions

Different ways to formulate an optimization problem

With words

Common for problems over discrete structures,
e.g. shortest path problem, minimum spanning
tree,...

With variables and equations

This is called *mathematical programming*.



in this
module

How should a can be designed?



What is the question?

What type of model seems appropriate?

Formulating the model

formulate the problem
mathematically

How minimize area
for given volume?

some variables

V volume

A area

r radius

h height

some equations

$$V = \pi r^2 h$$

$$A = 2\pi r^2 + 2\pi r h$$

the optimization model

$$\min 2\pi r^2 + 2\pi r h$$

Objective function

when

$$\pi r^2 h = 1$$

$$r \geq 0$$

$$h \geq 0$$

Constraints
describe
the feasible set

Note: only r and h appear as variables in the optimization problem!

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With variables and equations

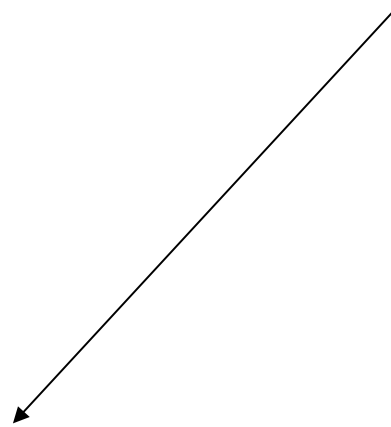
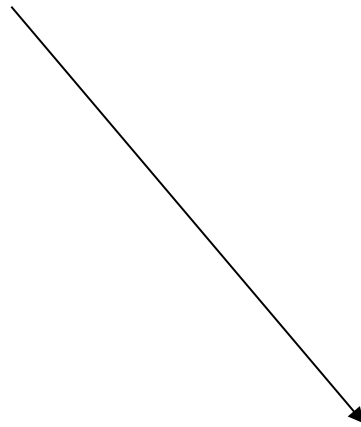
This is called *mathematical programming*.

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What is constrained optimization?

system of equations

unconstrained
optimization

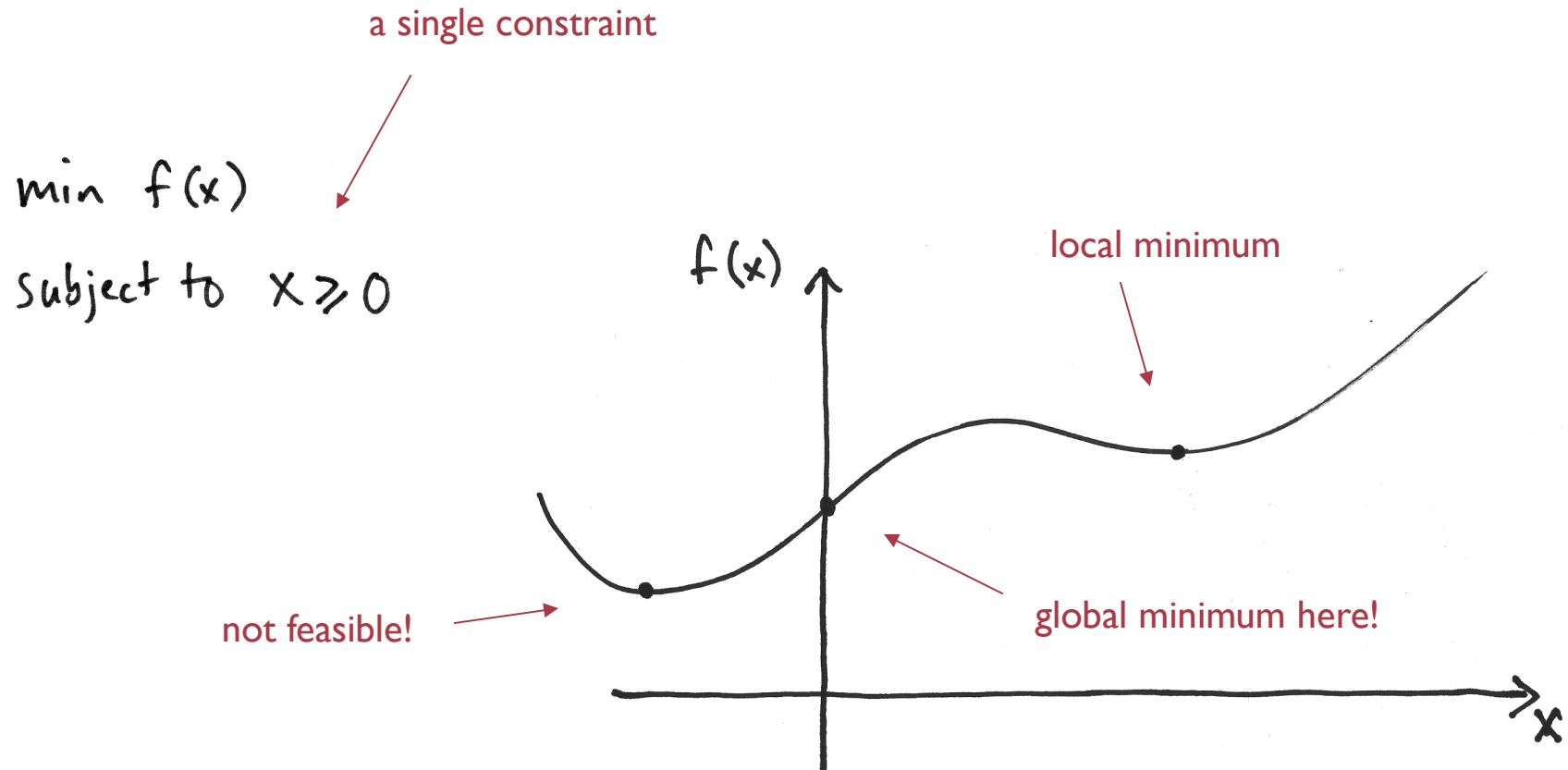


constrained optimization

you don't have to have as
many equations as
variables!

A powerful combination! But usually more difficult to solve than each part separately.
Fortunately there are many nice algorithms!

Why is it more complicated with constraints?



Now imagine many variables and constraints...

Important mathematical characteristics of optimization problems

- unconstrained/constrained
- linear/nonlinear
- continuous/discrete

Most common: linear and constrained.

Most important model & algorithm: linear programming.

Creating a mathematical optimization problem

1 Define variables: how can a solution be represented?

2 Write equations with the variables.

3 Define the objective function.

I Setting up the variables

How can a solution be represented?

Different kinds of variables:

- continuous
- continuous nonnegative
- discrete (categorical...)
- binary

Distinguish between

- variables and parameters!
- math variables and programming variables!

2 Setting up constraints

Much like setting up a system of equations.

But you can do more:

- You can have fewer equations than variables!
- You can use inequalities!
- You can have an objective function that determines which of all possible solutions you are interested in!

3 Define the objective function

How can you calculate the quality of a solution if you know it?

Do you wish min or max?

Note

Modelling the problem as an optimization problem

Solving the problem

- Two different things!

Simple assignment

tasks

persons

1	3	5	1
4	5	3	2
7	4	6	9
8	4	7	3

Simple assignment model

$$\min c_{11}x_{11} + c_{12}x_{12} + \dots + c_{44}x_{44}$$

a constant parameter

a variable

subject to

$$\begin{cases} x_{11} + x_{12} + x_{13} + x_{14} = 1 \\ \vdots \end{cases}$$

$$\begin{cases} x_{11} + x_{21} + x_{31} + x_{41} = 1 \\ \vdots \end{cases}$$

$$x_{ij} \text{ binary}$$

a binary integer linear program (ILP)

A common simplification

$$x_{ij} \text{ binary} \implies 0 \leq x_{ij} \leq 1$$

ILP

LP!

Easier to solve but can cause other difficulties

Linear Programming

- Special case of mathematical programming with:
 - linear objective function
 - linear constraints
- Fast algorithms exist that always find the optimal solution