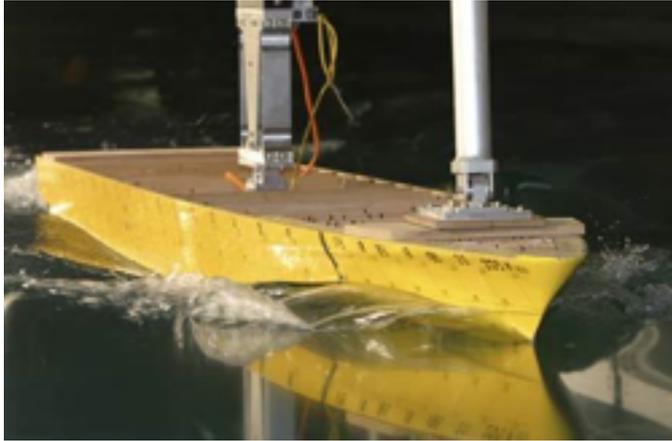


Reflections on modelling and problem solving

models

“Mathematical” models are just a special case



A model or a theory?

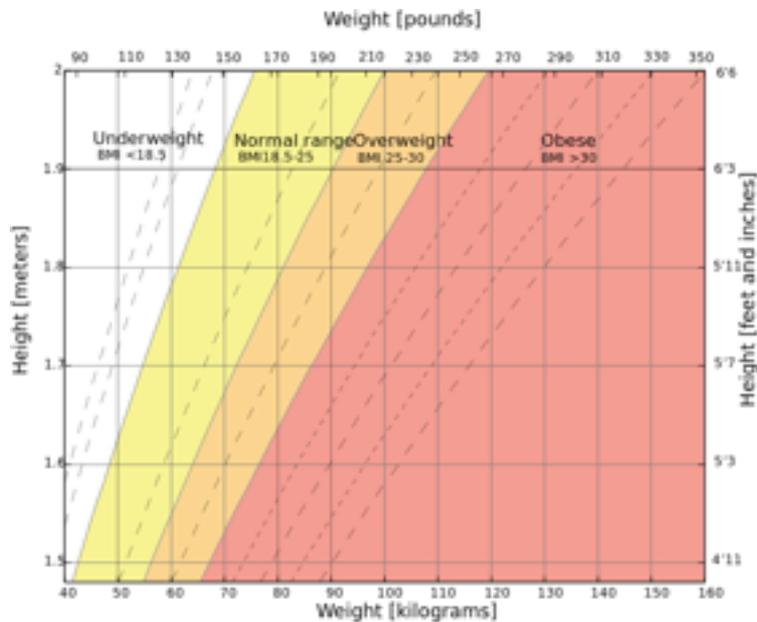
Model: intentional simplification often used for specific purpose. Not perfect but useful!

Theory: Often more general and comprehensive. A model that is assumed to be "correct".

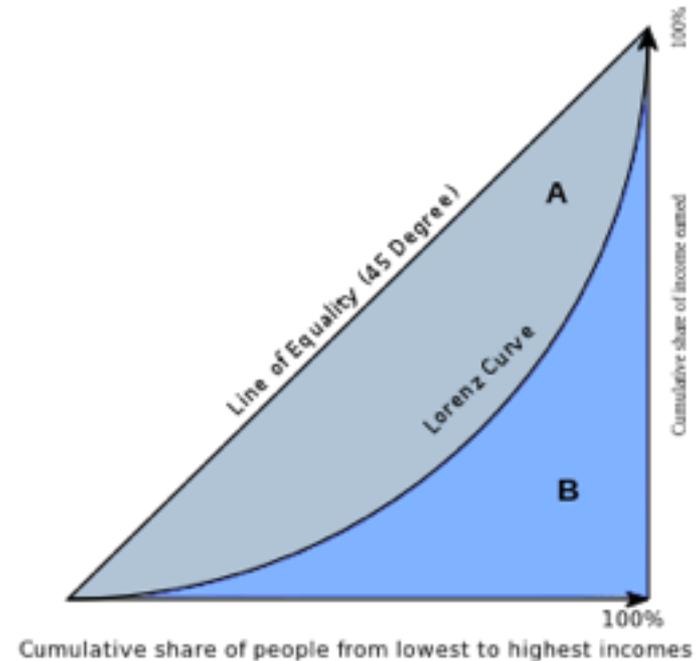
However, by modelling you may sometimes discover theories! (kepler, surprise)

Descriptive and prescriptive models

$$\text{BMI} = m/h^2$$



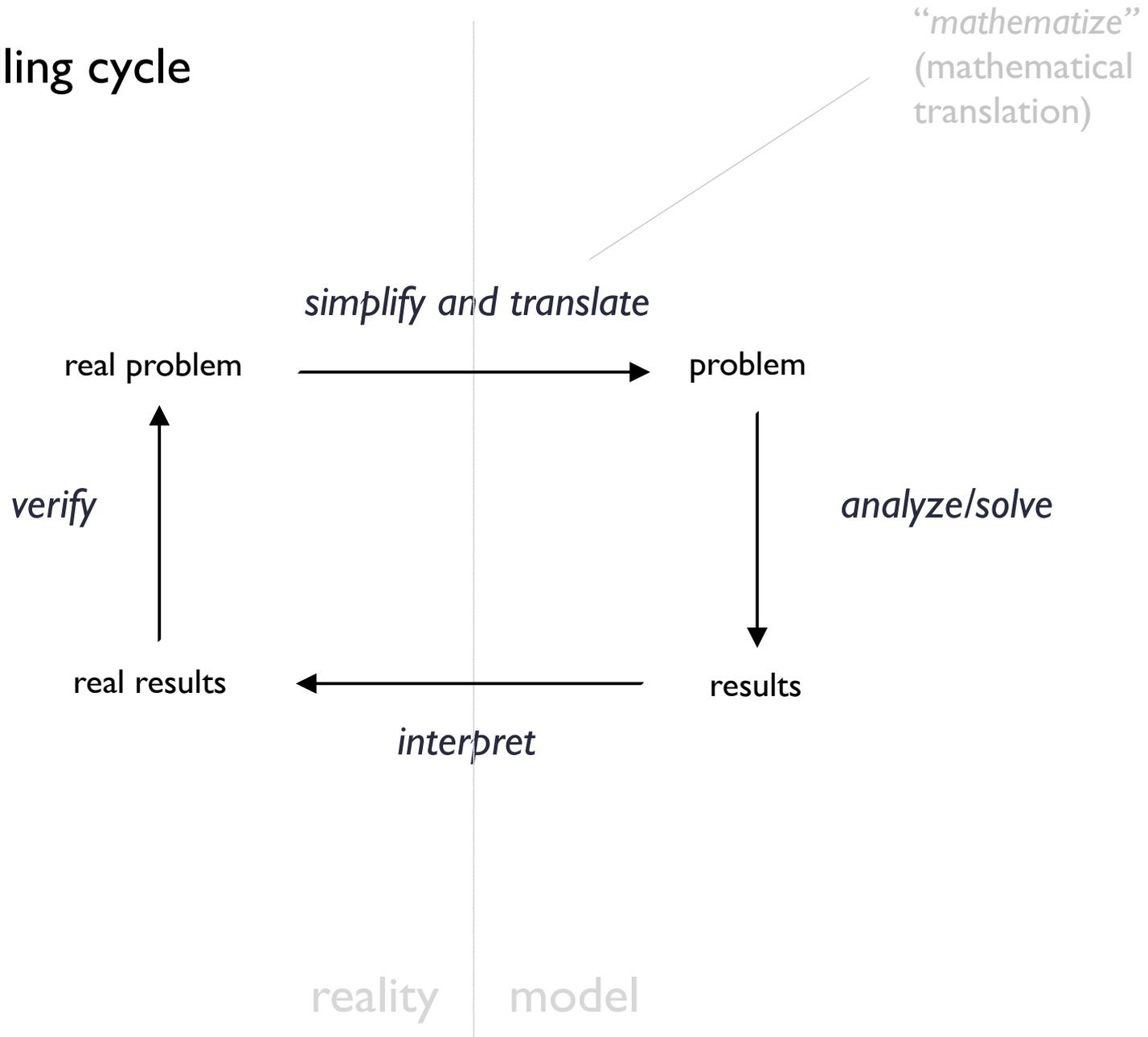
$$\text{GINI coefficient} = A/(A+B)$$



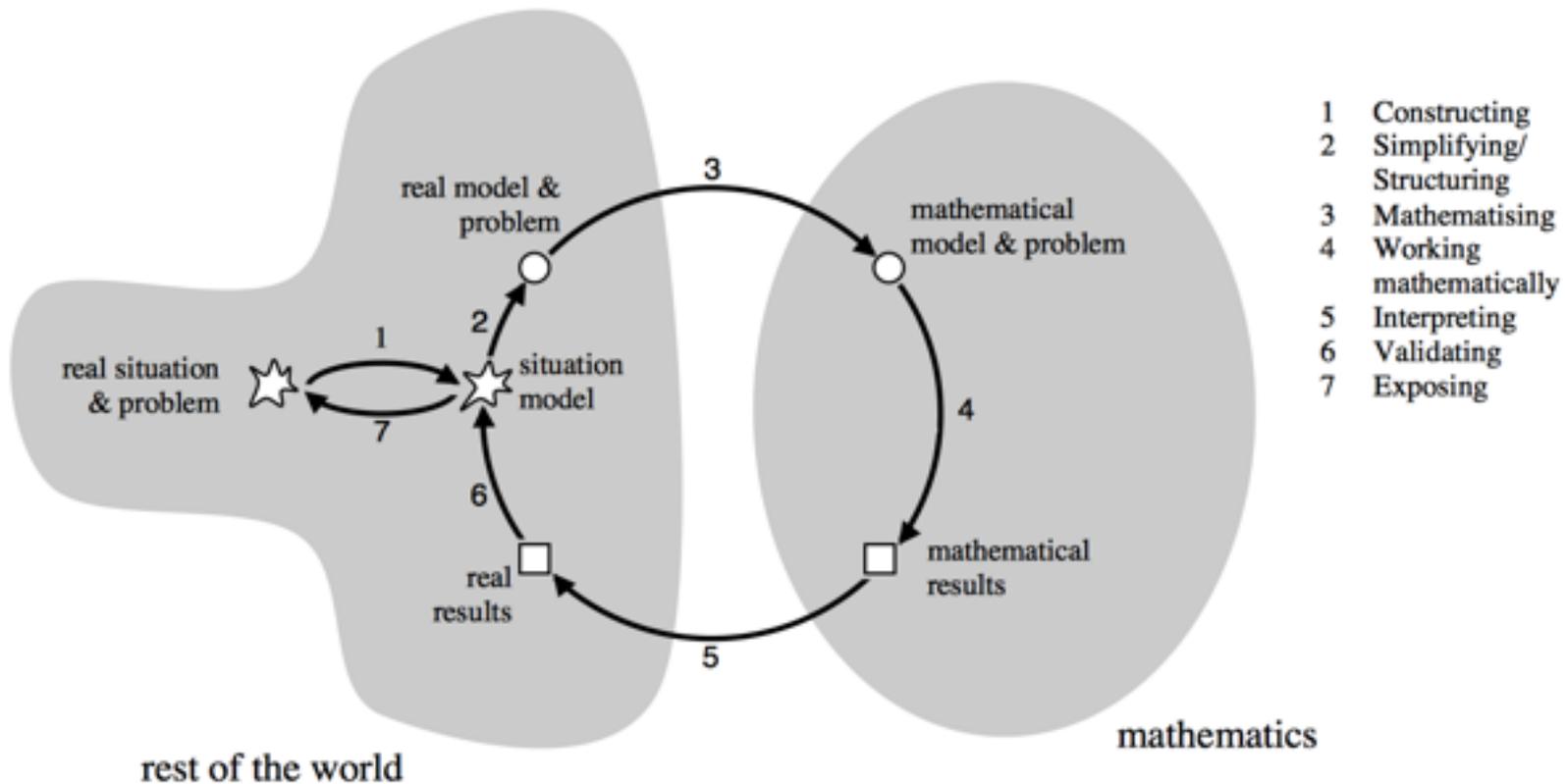
These models are artifacts, based on ideas of what we think are sensible concepts!

modelling

Modelling cycle



More elaborate modelling cycle



Blum and Leiß (2005)

First, the problem situation has to be understood by the problem solver, that is a *situation model* has to be constructed. Then the situation has to be simplified, structured and made more precise, leading to a *real model* of the situation...

What is the relationship between modelling and problem solving?

discuss!

What is the relationship between modelling and problem solving?

1. Modelling is a problem solving activity
2. Modelling is a part of your problem solving toolbox.
3. Modelling is related to simplification and moving between representations, which is important in all problem solving.

problem solving

Polya control strategy in four “phases”

1 understand the problem

2 make a plan (select the approach)

3 carry out the plan

4 look back

Each phase is associated with some standard questions and a set of heuristic strategies.

Some similarity to the modeling cycle (think about!)

Does not highlight the explorative nature of non-trivial problem solving

Heuristic strategies (Polya and earlier)

“rules of discovery and invention” (Polya)

“to understand a problem better or to make progress towards its solution” (Schoenfeld)

simplify, specialize, make example, split in parts, draw figure, find related problem, consider extreme cases, guess and check, systematically test ...

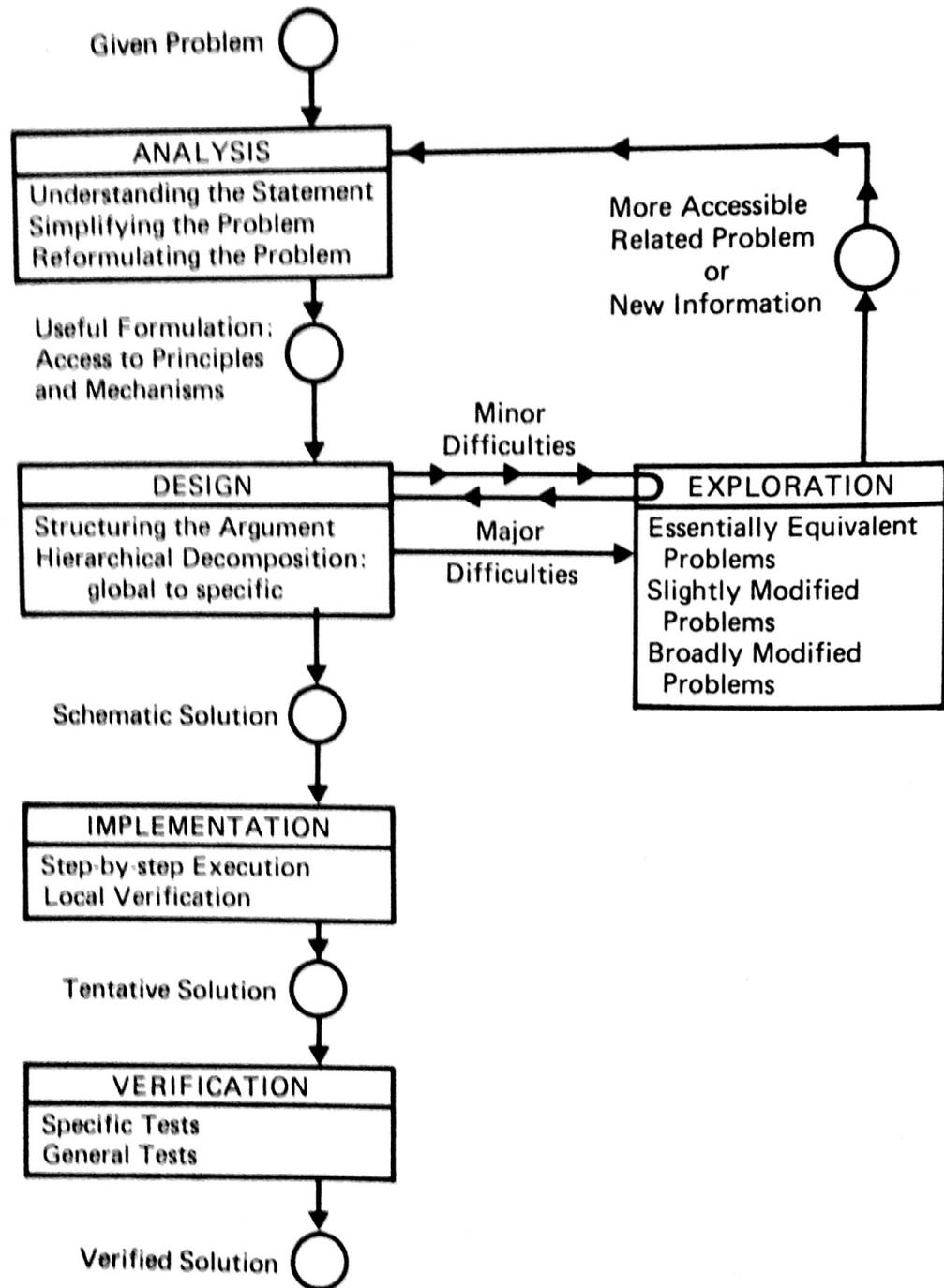
There are quite a few more...

Heuristics are not algorithms, but rather general descriptions of what you commonly do when you solve problems!

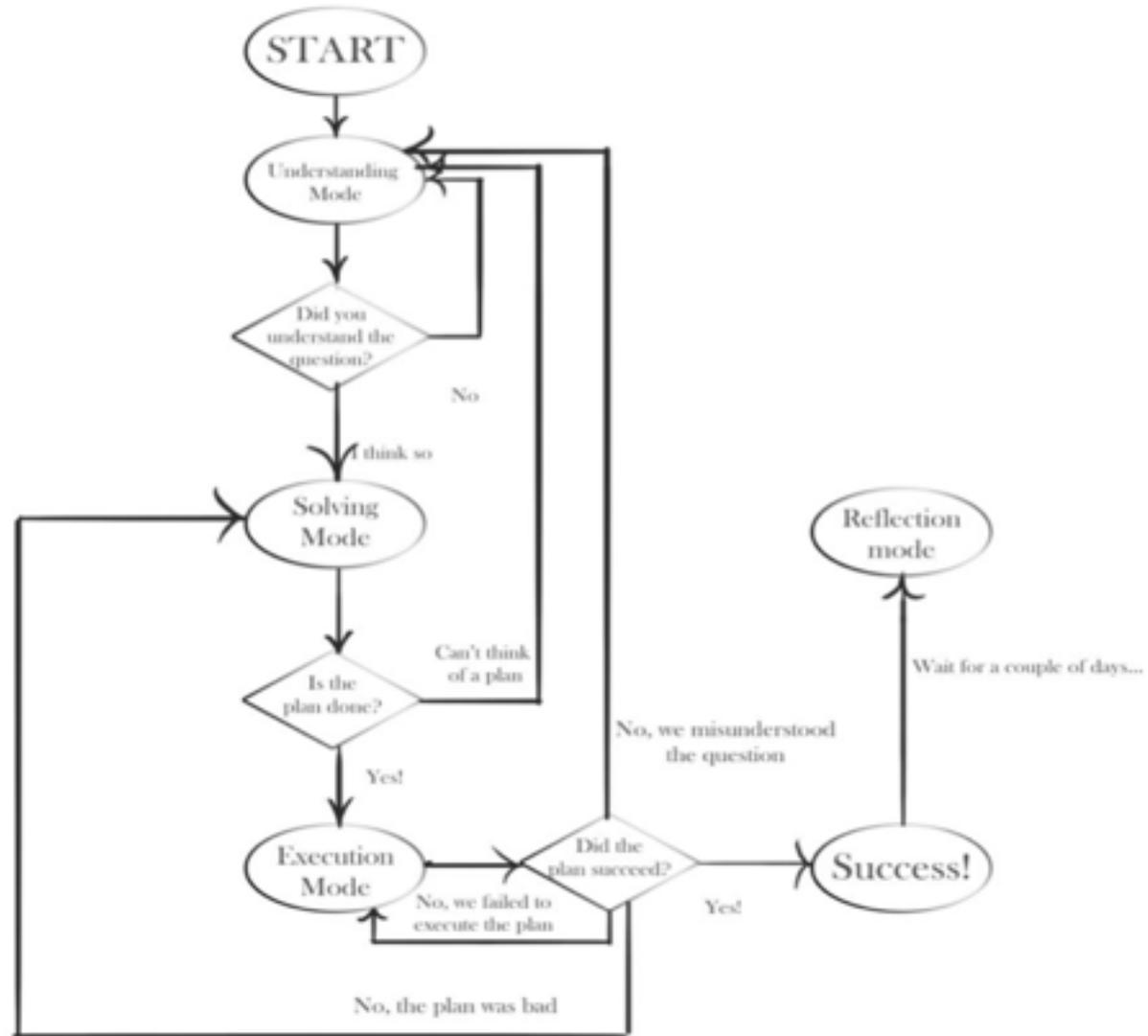
Think about some heuristics and how we have used them in different problems!
(also the modeling cycle...)

Schoenfeld control strategy

Makes the exploration more visible!



A student version! (from 2013)



Other student versions!

- Förstå problemet, vad som efterfrågas och hur svaret kan se ut
- Beskriv problemet med matematiska termer
- Välj modell och de variabler och restriktioner som behövs (planera hur man skall ta sig an problemet)
- Gå konstant framåt genom att lösa mindre delar av problemet, enklare eller specialfall
- Kontrollera om slutsatsen man nått är vad som efterfrågas och om det är rimligt
- Kan man nå någon djupare insikt utifrån vad man kommit fram till

Förstå!

Förenkla!

Gör om, gör rätt!

Initial steps (figure out the problem)

- Explain what the problem is.
- Understand by drawing figures.
- Attempt to set up the problem.
- Constants, variables, equations, and so on.

Explanation (examine and determine the problem components and scope)

- Discuss the problem further. ○ Possibly narrowing the scope or broadening the possibilities.
- Explore possibilities. This can possibly lead to simplifying the problem.
- Make necessary assumptions, either to enable progress or to restrict the scope of the problem.
- (Create necessary placeholder figures or write full figure texts.)

Result (find a method toward a solution)

- Create figures.
- Use tools.
- Write explanations and do calculations (with tools).
- Argue for or prove the correctness of the procedure.
- (Create necessary placeholder figures or write full figure texts.)

Final steps

- Draw conclusions, answering the problem.
- Correct by rereading the texts.
- Simplify the reasoning, if possible.

What is needed to be a good problem solver? (Schoenfeld)

Resources (knowledge of different kinds)

Heuristics (“tricks of the trade”)

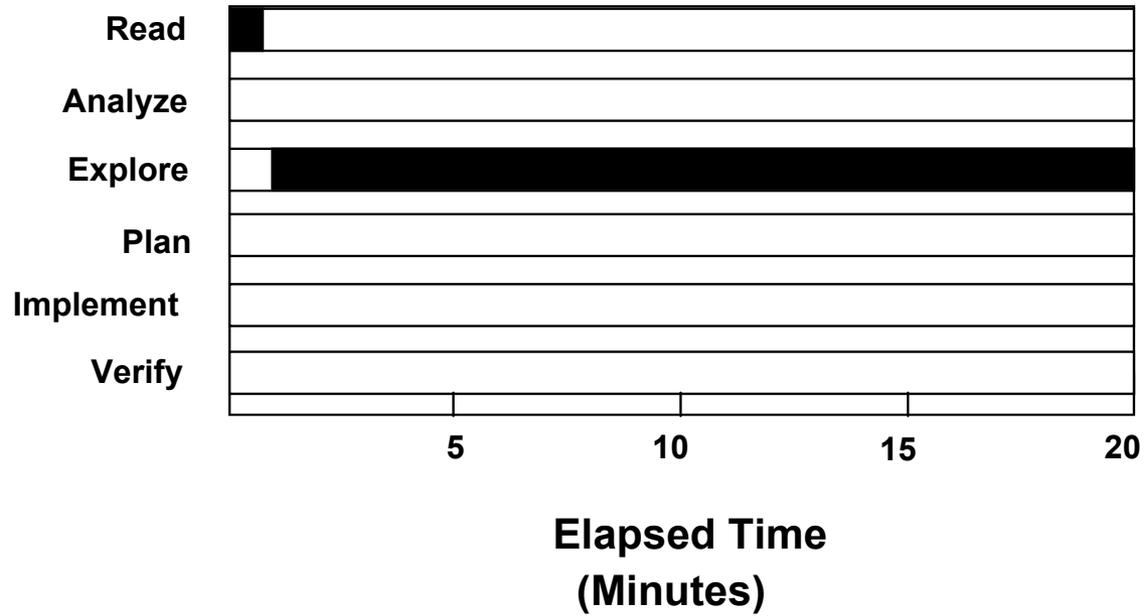
Self-regulation (monitoring and control, “self-awareness”)

Appropriate attitudes and expectations (your “belief system”)

discuss...

does this apply to all
complex tasks?

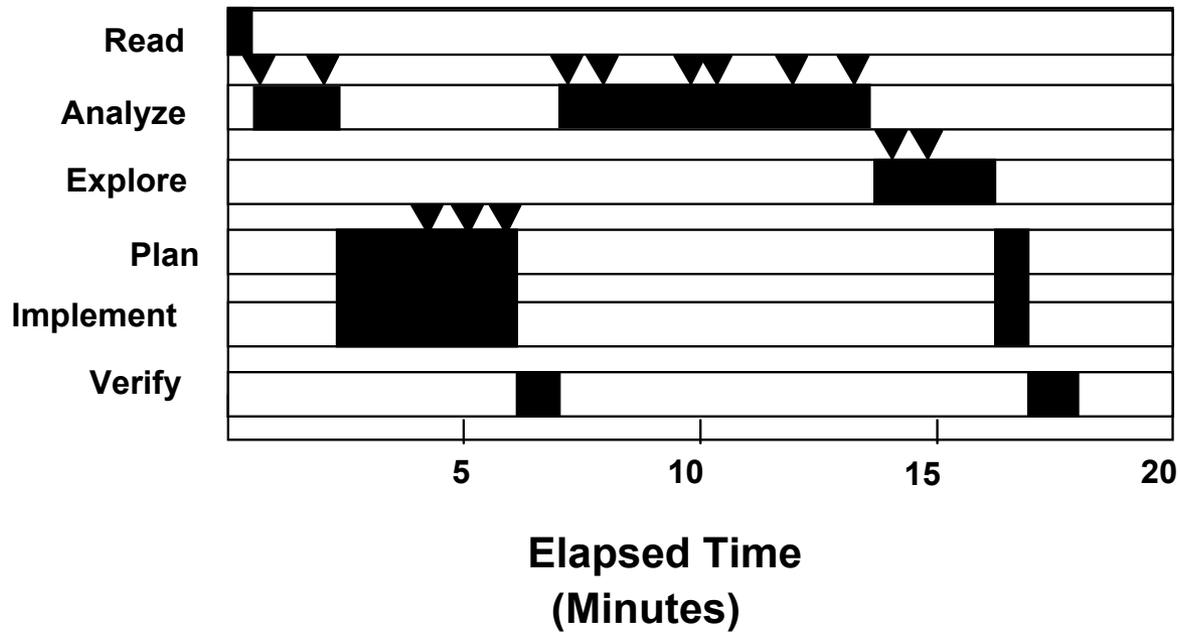
Activity



Time-line graph of a typical student attempt to solve a non-standard problem.

)

Activity



**Time-line graph of a mathematician
working a difficult problem**

about the course and some research

My starting point

Many software engineering and computer science students do not understand where and how to use mathematics.

Not even the mathematics they already know!

```
import java.awt.*;

public abstract class Animation
    extends java.applet.Applet
        implements java.lang.Runnable {

    protected Dimension d; // bitmap size
    protected Image im; // extra image for drawing
    protected Graphics offscreen; // the offscreen bitmap to draw in
    protected int delay = 100; // in milliseconds
    protected Thread animationThread;

    final public void init() {
        d = getSize();
        im = createImage(d.width, d.height);
        offscreen = im.getGraphics();
        initAnimator();
    }

    //final public void paint(Graphics g) {update(g);}

    final public void update(Graphics g) {
        paintAnimator(offscreen); // first draw offscreen to reduce flicker
        g.drawImage(im, 0, 0, this); // then put on screen
    }

    // To be implemented in subclass that does the actual drawing
    protected void initAnimator() {} // init for drawing routines
    abstract protected void paintAnimator(Graphics g); // the actual drawing
    will be here

    public void setDelay(int d) {delay=d;}

    public void start() {
        animationThread = new Thread(this);
        animationThread.start();
    }

    public void stop() {
        animationThread = null;
    }
}
```

What are the main challenges for students early in the course? (research project 2013)

Some modelling difficulties...

However, the main challenges are in problem solving:

1. understanding the problem

“The first week we really didn’t spend much time on trying to understand the problem and what the problem was. We just dove in and that kind of backfired sometimes...”

2. considering alternatives

“I don’t think we would have got stuck like we did if we ... tried to see different possibilities of getting forward, and not getting to an answer directly.”

- *I initially found exercise 2 very hard and wasted lots of time trying to solve it. It was mainly because I kept trying to solve it using one approach the entire time, but in the end the approach proved to be all wrong. To avoid this the next time, I will work on becoming less stubborn, and more open to trying new approaches.*
- *I also wasted some time on exercise 3 because I had completely misunderstood the problem (I thought the problem was basically the shortest path problem.). So I ended up wasting lots of time on the solving the wrong problem. ... I learned from this that you should truly understand the problem before you start solving it.*

Student in module 3, 2014

Attitudes and expectations are also important

Expect to try things out

Have open mind

Courage, patience, dare to use common sense

Self-confidence

“You need to have some courage to dare to try different things and not just sit there and wait...and if it doesn't work you must be prepared to discard it and try another one”

How the course works: inquiry based learning

Problems are starting points for learning.

Students explore and discover.

How the course works: cognitive apprenticeship

Authentic problems - learning is based on real practice

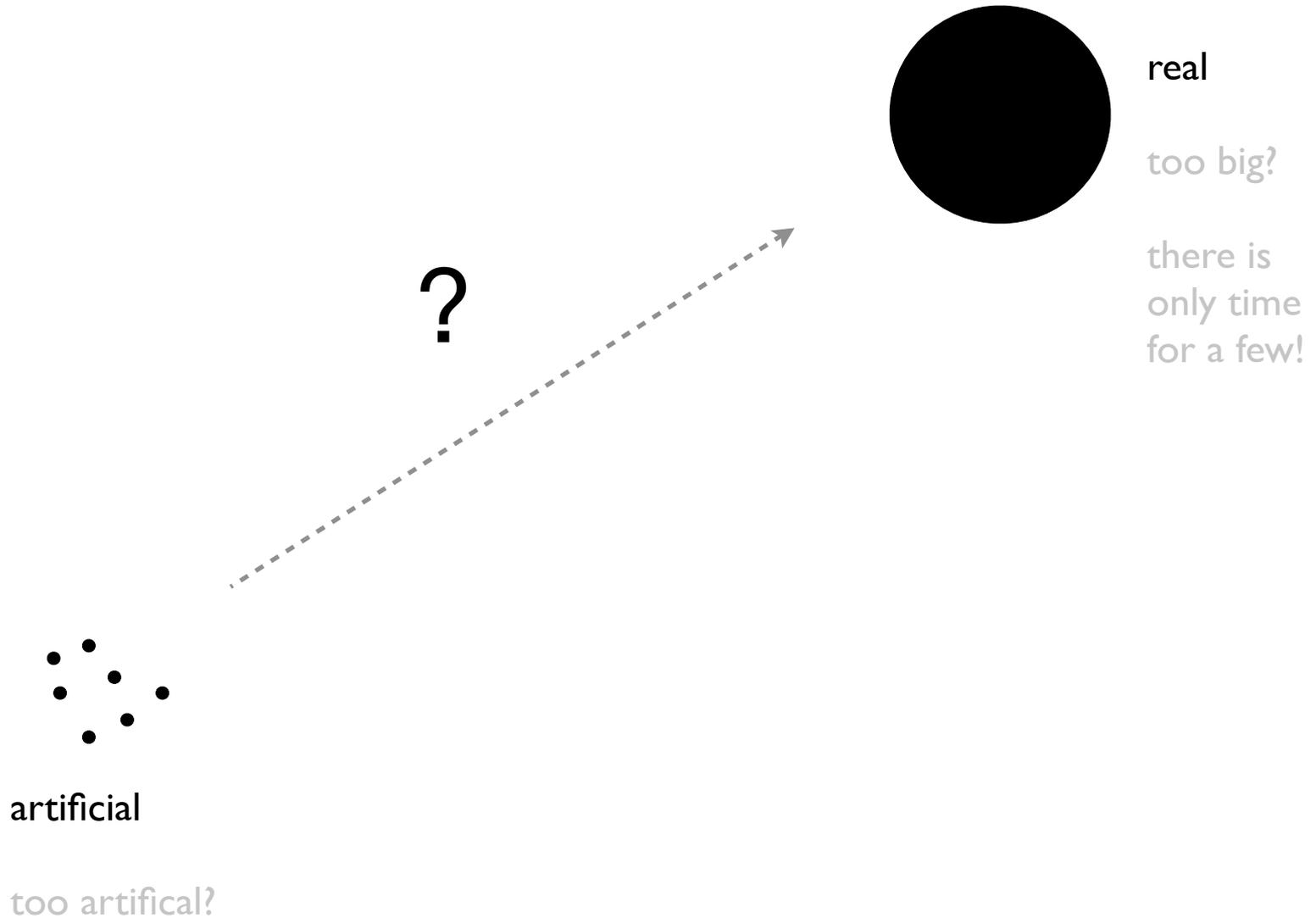
Make expert thinking visible - teacher shows good ways to think

Make student thinking visible - enables feedback and alignment with expert thinking

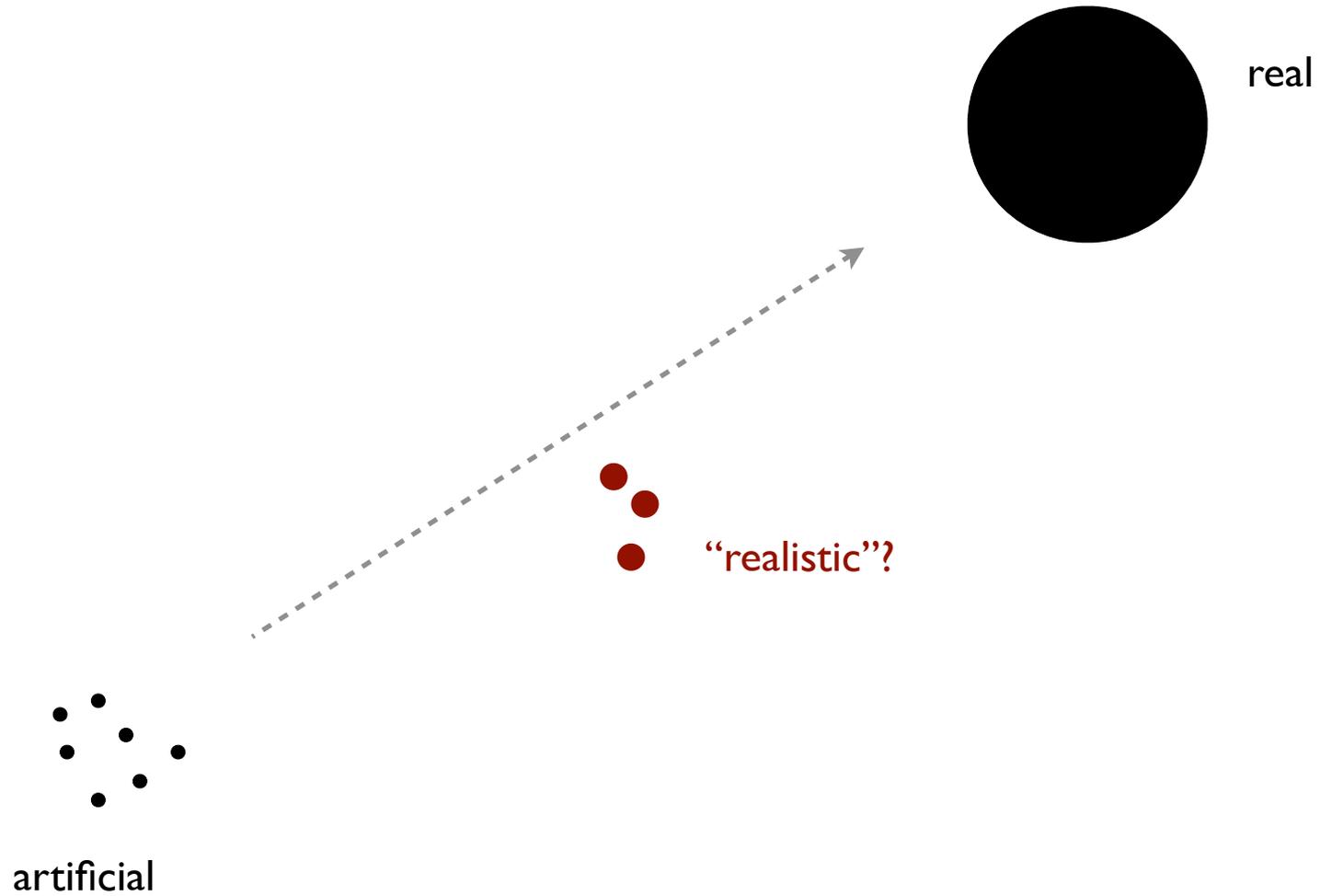
Cognitive apprenticeship can be applied in many areas when complex skills need to be taught, and to convey the experience of the teacher.

The course was originally designed intuitively based on experience, but we have found that cognitive apprenticeship is useful to explain how it works.

How design small “realistic” problems?



How design small “realistic” problems?



Real problems have meaningful answers!

1. The solution is directly useful in practice

2. The solution contributes to the understanding of some topic

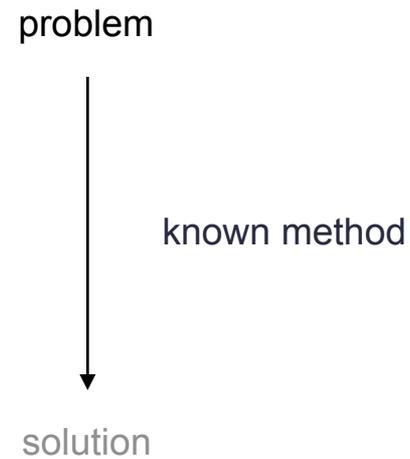
Real problems have meaningful answers!

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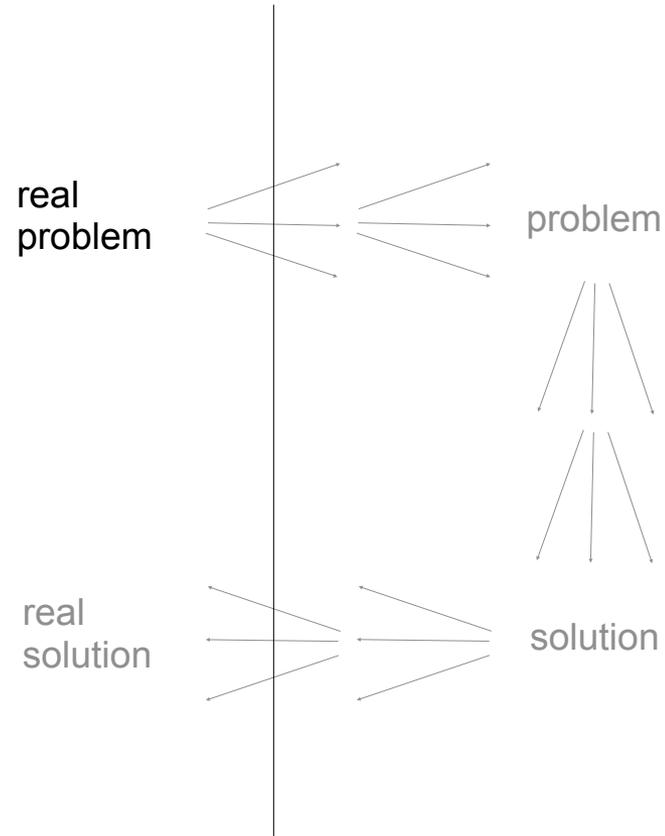
The problem should have a meaningful answer!

Common way of solving artificial problems



no modelling!
no problem solving!

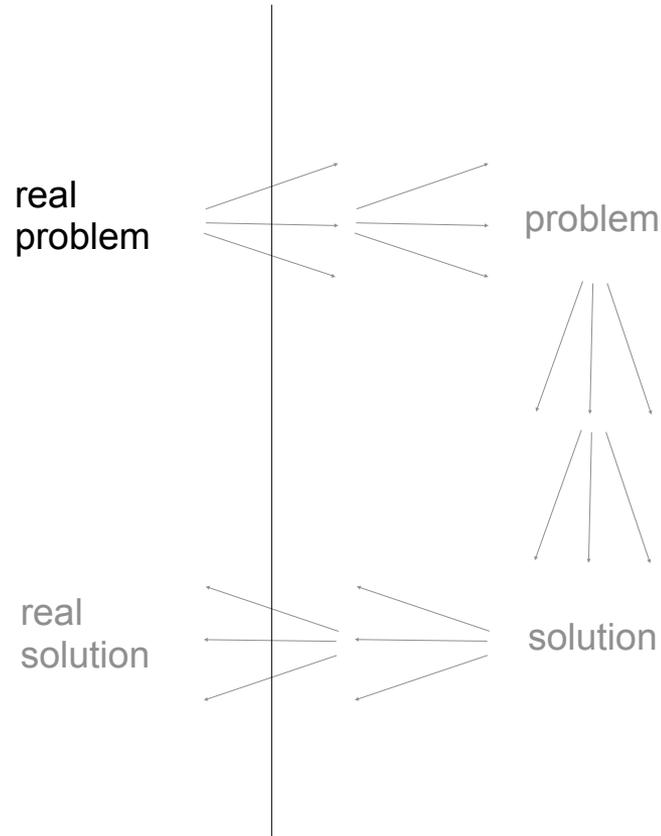
Skills needed for solving real problems



modelling and
problem solving!

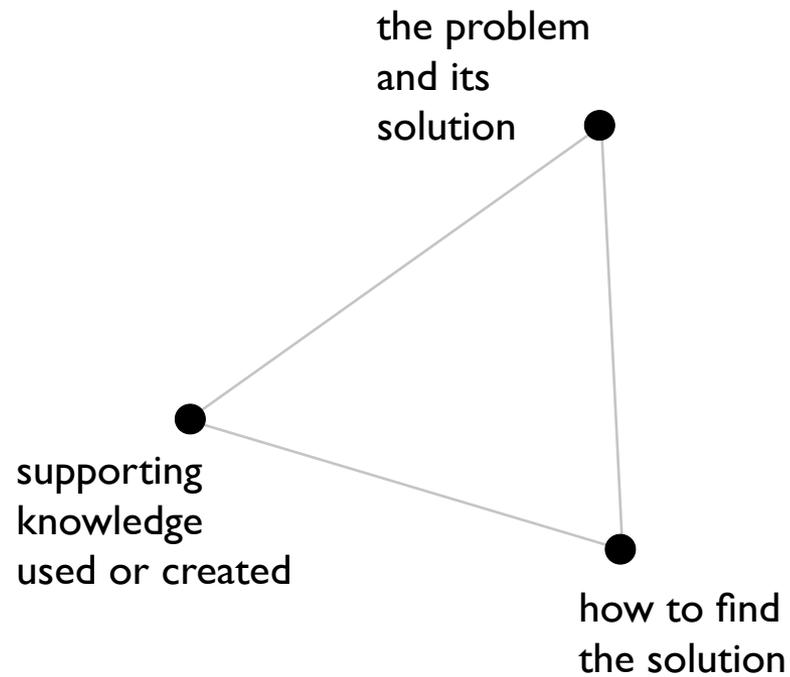
Skills needed for solving real problems

The problem should not be easy to understand!



The problem should be difficult to solve

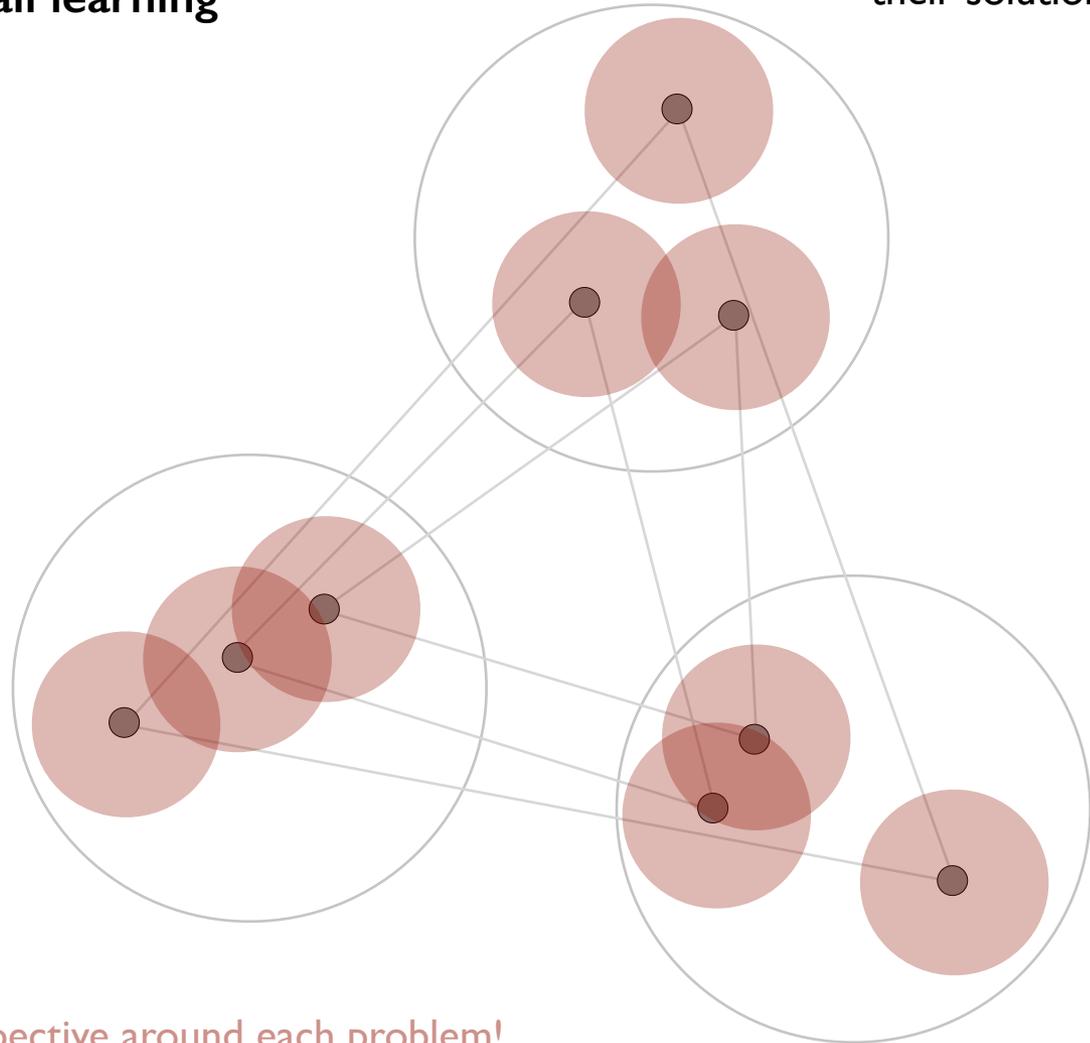
One problem has
several learning
opportunities!



Create a significant variation in all learning dimensions!

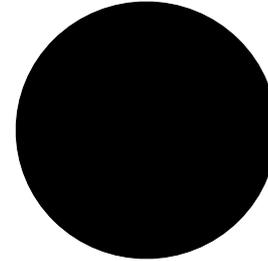
world of problems and their solutions

supporting knowledge
(concepts and methods)



also create a perspective around each problem!

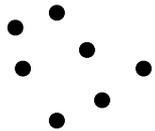
The course provides a middle ground between traditional courses and projects/thesis projects.



real



“realistic”



artificial

